Quasi-linear Time Algorithm for Dynamical Boundary Integral Equation Methods

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In the dynamic rupture simulation, the stress is problematically diverging around the moving rupture fronts. To deal with such stress without super-fine meshes, an analytic and accurate method, the dynamical boundary integral equation method (dynamical BIEM) have been frequently adopted in earthquake sciences [Tada 2009]. However, though we use the dynamical BIEM, the numerical cost is still huge. When in nonplanar problems, the cost of the naive implementation can be intractable even with super computers [Ando 2016].

To overcome such cost problems of dynamical BIEM, we provide novel quasilinear time algorithms, FDP=H-matrices. Originally, the memory consumption and the time to evaluate the stress integral are of $O(N^2M)$, where N is the number of elements and M is the number of time steps. Those costs are impressively reduced to be of $O(N\log N)$ by FDP=H-matrices (Fig. 1). It resolves the cost problem in dynamical BIEM.

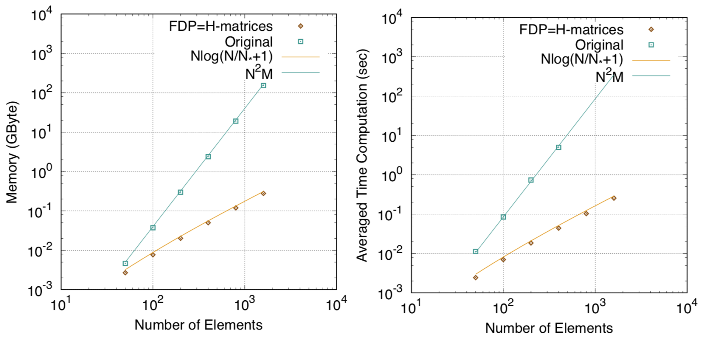

Figure 1. The costs of dynamical BIEM measured in a planar crack problem. FDP=H-matrices reduce the original $O(N^2M)$ costs to $O(N\log N)$ costs.

In this presentation, we show the outline of FDP=H-matrices and the performance evaluation. The cost reduction of FDP=H-matrices is based on the fact that the dynamical kernel obeys the geometrical spreading along the wavefronts [Ando, 2016]. In order to set the wavefronts numerically, FDP=H-matrices utilize the fast domain partitioning method (FDPM) [Ando et al, 2007; Ando, 2016]. The kernel along wavefronts is then expanded with the hierarchical matrices (H-matrices) [Hackbusch, 1999]; in the H-matrices, the expansion is done numerically and works well for FDP=H-matrices. Combining this simple combination with newly devised other algorithms to improve the accuracy, we now construct FDP=H-matrices; they also provide sufficient accuracy even for non-planar crack problems (Fig. 2), as detailed in presentations.

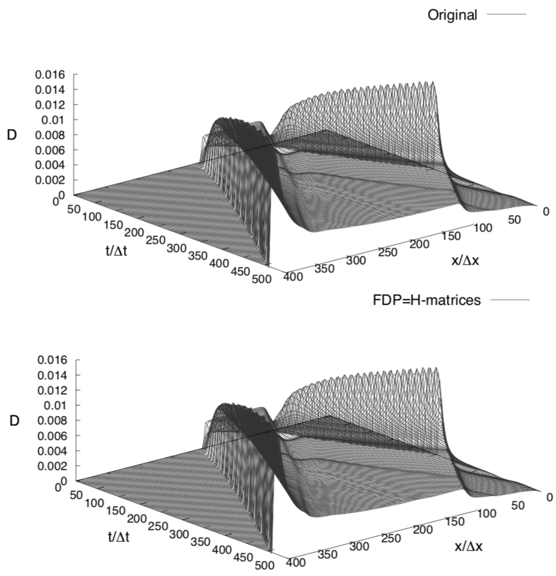

Figure 2. An example of simulated dynamic rupture without or with FDP=H-matrices; D is the slip rate, t is the time, & x is the position along a fault. The original solution is safely reproduced with FDP=H-matrices.